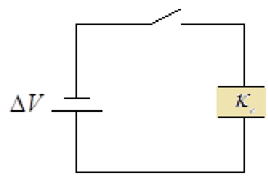
Homework 7: Capacitors ***due 5/14***

Problem 1. Hey look at that capacitor down there. It's one of those parallel plate-types ($A = 50cm^2$, $d = 1mm$, $\kappa_e = 500$). And the battery has a potential difference of 120V. Now say I flip the switch…



(a) What's the capacitor's capacitance?

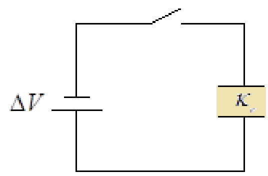
(b) What's the charge on the capacitor?

(c) What's the energy stored?

(d) What's the electric field within the capacitor?

(e) What's the force between the plates?

Problem 1′. Now say I keep the capacitor connected to the battery while I replace the $\kappa_e = 500$ dielectric with a $\kappa_e = 1000$ dielectric.



(a) What would be the new capacitance?

(b) Immediately after insertion, what would be the voltage across the capacitor?

(c) A 'long' time after insertion what would be the potential difference across the capacitor?

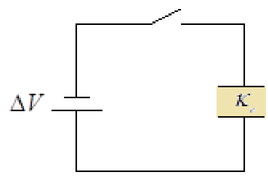
(d) What would be the new charge on the capacitor?

(e) What would be the new potential energy?

(f) What would be the new electric field?

(g) What would be the new force?

Problem 1´´. Now suppose that instead of inserting the new dielectric, while the capacitor was connected to the battery, I had disconnected the capacitor first, and then inserted the new κ_e = 1000 dielectric.



(a) What would the new capacitance be?

(b) What would be the new charge?

(c) What would be the new potential difference across the plates?

(c) What would be the new potential energy?

(d) What's the new electric field within the capacitor?

(e) What's the new force on the plates?

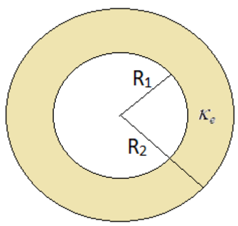
Problem 2. Before the advent of advanced dielectric materials, the only way to store a lot of charge was to have a 'big' capacitor (or a whooooole bunch of small ones). Say we wanted to store 1C of charge on a parallel plate capacitor with side lengths L, plate separation d, and air in between. We'd like to keep L as small as possible.

(a) Should we increase or decrease d, to keep L as small as possible.

(b) Decreasing d, increases E, and brings us closer to the dielectric breakdown strength of the air. If we reduce d to the point where the field between the capacitor plates is the dielectric breakdown strength of air (E = 3MN/C), what would L have to be to store 1C of charge?

(c) Now suppose we ditch the air for a 'super' dielectric $\kappa_e = 2\times10^6$, with a breakdown strength of 12MN/C. What would L have to be to store 1C of charge?

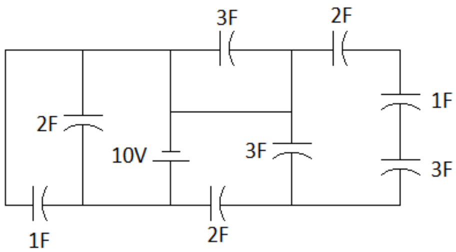
Problem 3. In class, we derived an expression for the capacitance of a parallel plate capacitor. (a) Derive the formula for a spherical capacitor. Take it to have inner radius R_1, outer radius R_2, and filled in between with dielectric κ_e. Should get $4\pi\kappa_e\varepsilon_0 R_1 R_2/(R_2 - R_1)$.



(b) Show that if we keep the distance between the plates, $R_2 - R_1$ constant, but make the radii very large, that this formula reduces to the parallel plate formula: $C = \kappa_e A\varepsilon_0/d$. This is why the parallel plate capacitor arrangement is more applicable than one might think.

Problem 4. So….we defined the equivalent capacitor to be the one which stores the same change at the same voltage as the network of capacitors it replaces. So here's a question, does the equivalent capacitor also store the same *energy* as the capacitor network it replaces. Prove that it does, or doesn't, for the parallel and series combinations. You'll find the formulas $PE = (1/2)C\Delta V^2 = Q^2/2C$ to be useful.

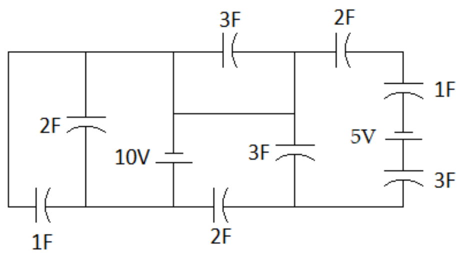
Problem 5. Consider the following network of capacitors. (a) Determine the charge on each one.



(b) If the battery were disconnected and some device, like a motor, were connected to the circuit in its place, how much charge would flow through the device?

(c) How much energy would be delivered to the device?

Problem 6. Consider the following network of capacitors (same as the old network, except for addition of a battery). (a) Determine the charge on each one. Might want to combine in series/parallel as much as possible before doing Kirchoff's equations.



(b) How much energy is stored in all the caps?

Problem 7. Sometimes, in lab, when setting up a capacitor network, you (*I* would *never*) forget to neutralize all the capacitors first, which makes the numbers you get all screwy. Consider the following arrangment, just before you connect the capacitors up to the circuit. Determine what the charges on the capacitors will be after you've connected them to the circuit, and the charges have settled. You'll want to use Kirchoff's laws. KCL will have to modified a little in this circumstance.